A Mixed Integer Programming Model to Control a One-Warehouse/*N*-Store Supply Chain

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Abstract

The current research work proposes a new inventory control system for a one-warehouse *N*-store supply chain facing a non-stationary demand. The supply chain uses a center unit (warehouse) to control all operations. The single product is of concern and the optimization of its flow is of interest. The new control system begins with solving a proposed mixed integer programming model to obtain the optimal inventory/distribution plan describing optimal shipping quantities and shipping periods. The one-period rolling horizon strategy is then adopted such that only the first period of the optimal plan is implemented. The performance of the proposed control system is evaluated against that of the echelon-stock *R*,*s*,*S* control policy, where *R* is a periodic review interval, *s* is a reorder point, and *S* is an order-up-to level, at various values of fill-rates. The numerical experiments show that the proposed control system outperforms the echelon-stock *R*,*s*,*S* control system for all ranges of system fill-rates. It is because the new system uses a sophisticated decision, which is based on the optimal plan considering the most up-to-date information of the entire system.

Keywords: Inventory/Distribution Plan, One-warehouse/Multiple-stores, Supply Chain, and Mixed Integer Programming

1. Introduction

The inventory control problem in supply chains is very complicated due to combination of several components, e.g. supply chain structures, coordination levels, information sharing processes, Bullwhip Effect (Forrester 1961). Adopting a classical inventory control system (*s,Q*, *s,S*, *R,S*, *R,Q*, etc.) based on objectives of a single firm is not the best strategy because of lack of concern of other supply chain members. The inventory control policy adopted by each entity always affects the inventory replenishment process of the upstream entity. Thus, for the supply chain management point of view, all entities should be planned and/or controlled simultaneously to obtain good control parameters and low inventory costs.

So far, much research has tried to determine suitable control parameters or improve basis decisions of classical inventory control systems adopted by supply chains. It normally aims to minimize the sum of inventory and transportation costs. Ganeshan (1999) proposed an approach to determine the near-optimal control parameters of the installation-stock *s,Q* system for a one-warehouse *N*-retailer supply chain. Yoo et al. (1997) proposed two effective order planning methods based on the installation-stock *s,Q* and *R,S* systems for a 1-CDC/*N*-RDCs network. Abdul-Jalbar et al. (2003) obtained inventory control parameters (replenishment times and reorder quantities) by solving various types of mathematical models concerning holding and ordering costs for *N*-retailer network. Axsäter (2003) proposed a technique considering holding and backordering costs to approximate optimal reorder points for a *N*-retailer network. Ryu and Lee (2003) determined order quantities and reorder points by solving a mathematical model concerning ordering costs, holding costs, and shortage costs under the installation-stock *s,Q* system. Yokoyama (2002) determined the target inventory and shipping quantity for a multi-DC/multi-Retailer model operating under the installation-stock *R,S* system. Tagaras (1999) calculated the Order-up-to quantities for a *N*-retailer network operating under the installation-stock *R,S* system. Optimal stock levels in general divergent networks under the echelon-stock *R,S* system was studied by Heijden (2000).

Some research including this paper focuses on proposing new inventory control systems and then compare their systems to a classical inventory control system. Yoo et al. (1997) proposed an improved Distribution Resource Planning (DRP) method using concepts of installation-stock *s,Q* and *R,S* systems. Ganeshan et al. (2001) studied two inventory control systems, namely, DRP and Reorder Point systems in a four-echelon network. A *s,S* system, where *s* and *S* vary with states, (SMART *s*,*S* system) was proposed by Giannoccaro and Pontrandolfo (2002). A three-stage serial supply chain is formulated as a SMDP (Semi-Markov decision process) model and then solved by SMART (Semi-Markov average reward technique) algorithm. Wang et al. (in press) proposed just-in-time distribution requirements planning (JIT-DRP) which aims to pull material through a multi-warehouse/multi-retailer supply chain effectively.

In this paper, a new inventory control system is proposed. It aims to optimize the product flow throughout a one-warehouse/multi-store supply chain. It controls each member by the optimal inventory/distribution plan operating under a one-period rolling horizon planning strategy. The optimal plan is obtained by solving a proposed mixed integer programming model. The objective function is to minimize the sum of ordering, holding, in-transit holding, transportation, and lost-sale costs.

The remainder of this paper is organized as follows. Section 2 describes a supply chain model. The new control system and safety stock policies are presented in sections 3. The experimental design is shown in section 4. Section 5 discusses the experimental results. Finally, the results are concluded in section 6.

2. The supply chain model

The concerned supply chain consists of one warehouse and *N* stores facing a non-stationary demand. The mean of the demand immediately changes from a low level to a high level for a certain interval and immediately returns to the low level. This demand behavior is commonly found in practice, for example, some businesses may have relatively high demands at the beginning periods of each month and relatively low demands at the remaining periods. The stores replenish their inventories from the warehouse, which in turn, replenishes its inventory from a vendor outside the concerned supply chain. The following assumptions are made to completely explain the model.

- The supply chain supplies a single product to customers.
- The customer demand follows Poisson distributions. The Poisson distribution is selected because it is not a rare case. One can observe that many businesses face Poisson-distributed demand such as businesses in which their customers wait in a long queue at the point-of-sale.
- The unsatisfied demand is accounted as lost-sale.
- All storage and transportation capacities are sufficient.
- Lateral transshipments between stores are not allowed.
- The model considers ordering, holding, in-transit holding, transportation, and lost-sale costs.

It is obvious that the system fill-rate and lost-sale cost depend on the supply ability of the supply chain. Unsatisfied demand, which results in low system fill-rate and high lost-sale cost, might occur if products are sent to a wrong place or in a wrong time. Nevertheless, filling the system with a large amount of inventories, which can protect against the shortage problem, is not the best strategy because of undesired holding costs of excessive inventories. Thus, it is interesting to optimize the product flow throughout the concerned supply chain such that the system can give the desired system fill-rate at the lowest total cost. In this paper, the supply chain is controlled by the proposed mixed integer programming model presented in the next section.

3. The mixed integer programming model

The mixed integer programming model to control the supply chain model is formulated as follows.

Indexes

t time index (1 . . *T*)

i node index (0 . . *N*), The warehouse has index *i* = 0, the store has index *i* > 0

Decision Variables

T_i^t shipping quantity from a source to node *i* at the beginning of period *t*

IB_i^t beginning inventory at node *i* in period *t*

IE_i^t ending inventory at node *i* in period *t*

L_i^t lost-sale quantity at node *i* at the end of period *t* (note that $L_{i=0}^t$ = 0, $\forall$ *t*)

B_i^t binary variable representing whether an order is placed by node *i* in period *t*

Parameters

M a large positive number

A_i fixed ordering cost at node *i*

h_i holding cost per unit-period at node *i*

hit_i per period unit holding cost in transit from a source node to node *i*

g_i unit transportation cost from a source node to node *i*

sl_i^t unit lost-sale cost at node *i* in period *t*

k_i safety stock factor at node *i*

d_i^t customer demand of the product at node *i* in period *t*

σ_i^t standard deviation of demand at node *i* in period *t*

L_i constant transportation lead time for transportation to node *i*

$IE_i^{t=0}$ initial inventory at node *i*

The mixed integer programming model

Minimize total cost =

$$\sum_t \sum_i A_i B_i^t + \sum_t h_{i=0} IE_{i=0}^t + \sum_t \sum_{i>0} h_i \{(IB_i^t + IE_i^t)/2\} + \sum_t \sum_i hit_i T_i^t + \sum_t \sum_i g_i T_i^t + \sum_t \sum_i sl_i^t L_i^t \quad (1)$$

s.t.

$$IE_i^t \geq k_i \ \sigma_i^t \qquad \forall\ t, i \quad (2)$$

$$IB_i^t = IE_i^{(t-1)} + T_i^{t-L_i} \qquad \forall\ t, i \quad (3)$$

$$IE_i^t = \begin{cases} IB_i^t - \sum_{i>0} T_i^t & \forall\ t, i=0 \\ IB_i^t - (d_i^t - L_i^t) & \forall\ t, i>0 \end{cases} \quad (4)$$

$$M B_i^t \geq T_i^t \qquad \forall\ t, i \quad (5)$$

$$T_i^t\ ,\ IB_i^t\ ,\ IE_i^t\ ,\ L_i^t\ \geq \quad 0 \qquad \forall\ t,\ i \qquad (6)$$

$$B_i^t \text{ is binary} \qquad \forall\ t,\ i \qquad (7)$$

Equation 1 is the objective function. It aims to minimize the total cost comprising of ordering, holding, holding cost in transit, transportation, and lost-sale costs. Constraint 2 is the safety stock policy constraint. The ending inventory at each node must not be lower than the required safety stock level. Constraints 3 and 4 are inventory balance constraints. Constraint 5 is a fixed cost constraint. The binary variable B_i^t equals 1 if the shipping quantity T_i^t is more than zero. It assumes that transportation always occurs after an order has been placed from a destination node. Thus, the fixed ordering cost is always incurred when transportation occurs.

Solving the model, the planner obtains the optimal plan containing the optimal shipping quantity and inventory level at each node in each period. Since the mean value of customer demand is entered into the model. Perfectly following the optimal plan in every periods might not result in the real minimum total cost because the actual demand might be different from the mean value. Thus, in this paper, only the first period of the optimal plan is implemented. The state of the system is then updated and the optimal plan is regenerated with the horizon advanced by one period.

According to the model, there are two points to keep safety stock to hedge against the uncertainty. Thus, the safety stock policy is denoted by [K1,K2], where K1 and K2 represent the safety stock parameters at the warehouse and stores, respectively (K1 = $k_{i=0}$ and K2 = k_i , where *i* > 0). Due to limitation of the computation time, the safety stock factor k_i is restricted to integer numbers and assumed to be equal for all stores (when *i* > 0). To determine the most suitable safety factor policy [K1,K2], which can give the desired fill-rate and the lowest total cost. The following searching process is conducted: set K1 equals integer numbers (0, 1, 2, 3, …). For each value of K1, search for the integer value of K2 which gives the desired fill-rate and the lowest total cost by the simulation experiment presented in the section 4.1.

4. Simulation experiments

The experiments are divided into three main parts as follows:

4.1 Study on roles of safety stock factor k_i

Due to the safety stock policy searching process (see section 3), the values of K1 and K2 will be varied and the effects on fill rates and total costs will be analyzed.

4.2 Comparison between the proposed and *R*,*s*,*S* control systems

The effectiveness of the proposed control system is evaluated against that of the echelon-stock *R*,*s*,*S* control policy, where *R* = 1. The echelon *R*,*s*,*S* control policy is selected because it contains many characteristics similar to the IDP system. Those are a one-period review interval, variable lot sizing, and network-information utilization. The parameters *s* and *S* are determined by a manipulation process—hold all factors constant, vary *s* and *S*, and then run the simulation until the required fill-rate is obtained (for details see Whybark and Yang 1996).

4.3 Study on roles of the supply chain cost structure

The supply chain cost structure is changed by increasing the unit lost-sale cost by 20% while other cost parameters are kept constant.

Simulation details

The full factorial experiments are performed for two levels of the following parameters: the number of store (two or six stores), and fixed ordering cost A_i (high or low level). The simulation experiment covers four weeks with a first-week warm-up period. The related costs and system fill-rates are recorded from the second week to fourth week. It is repeated until the 95% confidence intervals of system fill-rates and total costs stay within 5% of their means. Table 1 shows all related parameters. Note that *Pc* denotes the product price. In the experiment, *Pc* is set to fifty baths.

TABLE 1: Related parameters of supply chains

Parameters	Value	Parameters	Value
$A_{i=0}$	20 Baht	hit_i, $\forall i$	150% of h_i
A_i, $i > 0$	Rnd(10-20)[1] Baht	g_i, $\forall i$	Rnd(0.3%-0.9%) of *Pc*
$h_{i=0}$	0.375% of *Pc*	sl_i^t, $\forall i, t$	50% of *Pc*
h_i, $i > 0$	Rnd(0.3%-1.2%) of *Pc*	L_i, $\forall i$	1 period

[1] Rnd(X-Y) denotes the number which is randomly drawn between X and Y.

The optimal plan is generated by the LINGO 8.0 to cover a 7-period planning horizon. Note that the planning horizon should be long enough such that the optimal plan contains at least two inventory replenishments at each node. Since the customer demand follows Poisson distributions, the associated standard deviations of demand at the stores and warehouse are equal to $\sqrt{\lambda_i^t}$ and $\sqrt{\sum_{i>0} \lambda_i^t}$, respectively, where λ_i^t denotes mean of demand at store *i* in period *t*. The initial inventory at the warehouse is set to $k_{i=0} \bullet \sigma_{i=0}^t$ and that at store *i* is set to $\{(L_{i=0} + L_i)\lambda_i^t + (k_i \ \sigma_i^t)\}$. For the non-stationary demand situation, mean of demand at low/high level is randomly drawn from the interval 5-15/25-75. The high level occurs at the 16th period until the 25th period. The variation of the fixed ordering cost A_i is considered in order to represent the lost-sizing policy. The high level of the fixed ordering cost A_i representing big lot size (approximately covering three or four periods of demand) is set as shown in Table 1. The low level representing small lot size (approximately covering one or two periods of demand) is set at 50% of the high level.

5. Results and discussions

5.1 Roles of safety stock factor k_i

The mean values of total costs and fill-rates are recorded and tabulated for all four cases of the two-level factorial experiment. An example of the simulation results for the one-warehouse/two-store supply chain operating under high rate of ordering cost is shown in Table 2.

TABLE 2: Total costs/fill rates with various safety stock parameters (An example**)**

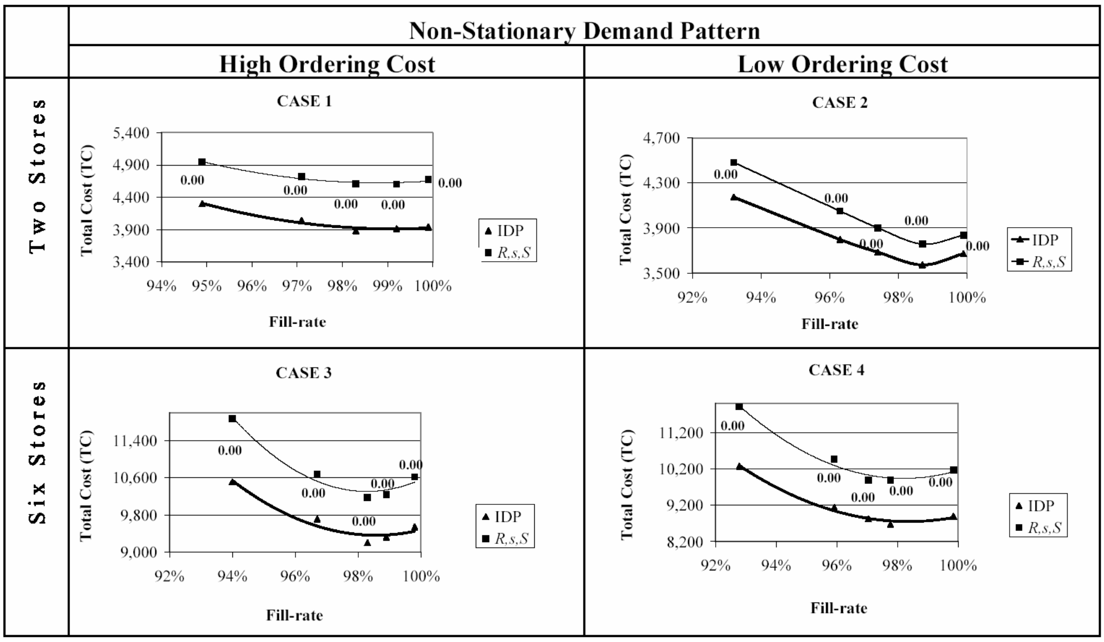
		Safety stock parameter at Stores, **K2**				
		0	1	2	3	4
Safety stock parameter at Warehouse, **K1**	0	2,164 **94.57%**	2,056 **97.28%**	1,986 **98.76%**	1,898 **99.56%**	2,010 **99.80%**
	1	2,183 **94.57%**	2,062 **97.62%**	1,996 **98.76%**	2,015 **99.56%**	2,024 **99.80%**
	2	2,177 **95.05%**	2,100 **97.38%**	2,013 **98.76%**	2,034 **99.56%**	2,044 **99.80%**
	3	2,219 **94.57%**	2,143 **96.68%**	2,030 **98.76%**	2,051 **99.56%**	2,062 **99.80%**
	4	2,237 **94.57%**	2,161 **96.68%**	2,034 **98.76%**	2,067 **99.56%**	2,080 **99.80%**

Note: fill rates are presented in bold type

For each row in Table 2, One-way ANOVA is applied to ensure the positive correlation between the system fill-rate and the safety stock parameter at stores (K2). Thus, safety stock at the stores always contributes to customer service level. For each column, one-way ANOVA is also performed and it is found that the safety stock parameter at the warehouse (K1) insignificantly affects the system fill-rate, but has a positive correlation with the total cost—K1 increases, the total cost increases. Note that the policy [0,3] is the best policy for the system fill-rate of 99.66% because it provides the lowest total cost (1,896) when comparing to other policies which also provide the same range of system fill-rate (99.66%)—the policy [0,3], [1,3], [2,3], [3,3] and [4,3]. The results for the remaining three cases (not shown here) are similar to the one in Table 2. Thus, all cases lead to a conclusion that the safety stock parameter at the warehouse (K1) should be zero and that at the stores (K2) affect both fill rates and total costs.

5.2 Comparison between the proposed and *R*,*s*,*S* control systems

Figure 1 shows the results. Five points of average total costs for both control systems are plotted against the average fill-rates. Note that five points of the proposed system are from the policies [0,0], [0,1], [0,2], [0,3], and [0,4], respectively. The bold numbers on each chart are p-value (observed significance level). Based on the results, there are a number of observations as follows.



Note: The values beside the graph line is the p-value of paired t-test.

FIGURE 1: Total costs and fill-rates obtained from the experiments

- From Figure 1, it is clear that the proposed system provides better results for all cases. Its average total costs are always lower than those of the echelon-stock *R*,*s*,*S* control system for all ranges of system fill-rate. The p-value of zero reveals that the difference is statistically significant.
- The ordering cost is varied to evaluate the effect of lot sizes on performance of both systems. From Figure 1, the proposed control system significantly outperforms the echelon-stock *R*,*s*,*S* control system for both levels of ordering costs (both small and large lots).
- The two-store supply chain represents a small supply chain while the six-store supply chain represents a large supply chain. It can be seen from Figure 1 that the proposed control system still significantly outperforms the echelon-stock *R*,*s*,*S* control system for both supply chain sizes.
- Based on the experiments, the proposed control system always provides at least 90% of system fill-rate (see Figure 1), even though the safety stock policy [K1,K2] is set to the worst case [0,0]. For the echelon-stock *R*,*s*,*S* control system, the system fill-rate can be lower than 90%, if the control parameters are set improperly (not shown here). Thus, the user must carefully set the control parameters of the echelon-stock *R*,*s*,*S* control system, otherwise the system performance may be unacceptable.

5.3 Results of the study on effects of cost structures

When the supply chain cost structure is changed by increasing the unit lost-sale cost by 20% and repeat all experiments, the comparison results on performances between both control systems are still unchanged. The graph showing total costs and fill rates of both control systems for only case 1 is presented as an example in Figure 2. It can be seen that the increase of unit lost-sale cost only affects the minimum point. The minimum point is shifted to the right (higher fill-rate) because the system needs more inventories to protect against the higher unit lost-sale cost. Therefore, the proposed control system outperforms the echelon-stock *R*,*s*,*S* control system for non-stationary demand at different cost structures.

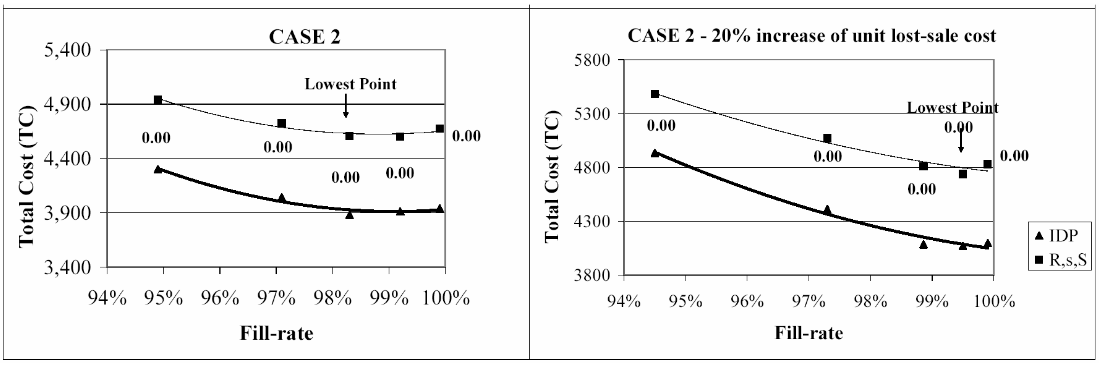


FIGURE 2: Effect of unit lost-sale cost on the lowest point of total cost

6. Conclusions

This paper aims to propose a new inventory/distribution control system for a multi-store supply chain in a non-stationary demand mode. The proposed model is developed to determine optimal inventory and distribution plan that minimize the sum of ordering, holding, holding in transit, transportation, and lost-sale costs. The optimum inventory/distribution plan is updated every period and only the first period is implemented. The proposed system is tested against the well-known echelon-stock *R*,*s*,*S* control system. Due to the experiment results, the new system outperforms the echelon-stock *R*,*s*,*S* control system for any ranges of system fill-rate. The new system is more effective because of its more the sophisticated decision which based on the optimal plan considering the most up-to-date information of the entire system. Future research should study a coordination scheme or a reward system (e.g. incentive mechanisms) to fairly allocate the benefits obtained from the proposed system between the warehouse and stores to encourage coordination.

7. References

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