Fuzzy Stochastic Data Envelopment Analysis (FSDEA) Modeling for Supply Chain Performance Evaluation

Varathorn Punyangarm[a,b], Patcharaporn Yanpirat[b], Peerayuth Charnsethikul [b], Saowanee Lertworasirikul[c] and Rawee Suwandechochai[d]

[a]Department of Industrial Engineering, Srinakharinwirot University (Ongkarak)
[b]Department of Industrial Engineering, Kasetsart University (Main Campus)
[c]Department of Product Development, Faculty of Agro-Industry, Kasetsart University (Main Campus)
[d]Department of Mathematics, Faculty of Science, Mahidol University

Abstract

The concept of traditional data envelopment analysis (DEA) model has been used for measuring relative efficiencies of a set of homogenous decision making units (DMUs) by comparing a target DMU with other DMUs that utilize the same multiple inputs to produce the same multiple outputs based on linear programming (LP) techniques. Unlike the traditional DEA model, the concept of supply chain management requires the performance of overall supply chain rather than the performance of each individual member. This paper applies the methodology of DEA model for evaluating supply chain performance (DEA-SC) and achieving the best practice. However, since the traditional DEA model is in the form of a LP model, the assumption of crisp deterministic inputs and outputs are required. This requirement is limited to applications of the traditional DEA model in real world problems with both variation in term of randomness and vagueness of input and output data. In this paper, the fuzzy stochastic DEA model for evaluating supply chain performance (FSDEA-SC) is proposed to handle simultaneously both randomness and vagueness. Two steps of transforming the FSDEA-SC model into the crisp deterministic DEA model for evaluating supply chain performance (CDDEA-SC) used in this paper are based on the concept of chance-constrained programming (CC) and the possibility approach.

Keywords: Chance-Constrained Programming, Data Envelopment Analysis, Supply Chain Management, Possibility theory

1. Introduction

The concept of supply chain management requires the performance of overall supply chain rather than each individual members. There are three reasons for measuring the performance of supply chain system. First, measuring the performance to provide the basis to understand the supply chain operations. Second, measuring the performance to monitor and manage the supply chain system through identifying the best practice supply chain operations. Finally, measuring the performance to provide directions of supply chain system improvement. (Zhu, 2003) A typical supply chain system can be presented in Figure 1 with four echelons.

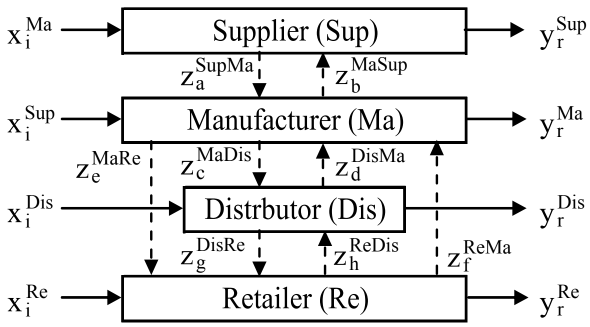

Figure 1 Typical Supply Chain System

From Figure 1, Supply chain system consumes m inputs and produces s outputs. Let DI^{Δ} for i = 1, …, m and DR^{Δ} for r = 1, …, s respectively represent the direct input and output subscript sets for a supply chain member Δ and x_i^{Δ} for $i \in DI^{\Delta}$ and y_r^{Δ} for $r \in DR^{\Delta}$ denote direct inputs and outputs, respectively. Since each such output also represents an input to associated supply chain member then only intermediate outputs are defined. Let z_a^{SupMa} for a = 1, …, A; z_b^{MaSup} for b = 1, …, B; z_c^{MaDis} for c = 1, …, C; z_d^{DisMa} for d = 1, …, D; z_e^{MaRe} for e = 1, …, E; z_f^{ReMa} for f = 1, …, F; z_g^{DisRe} for g = 1, …, G; z_h^{ReDis} for h = 1, …, H respectively represent the intermediate outputs from sup to ma, which is the intermediate input from ma to sup, ma to dis, dis to ma, ma to re, re to ma, dis to re and re to dis.

Since a supply chain system can be viewed as an integrated input-output system which involves direct and intermediate inputs or outputs of each supply chain member, then the DEA model which was first introduced by Charnes et al. (1978) can be adopted to measuring performance of the system. Since the traditional DEA model is build based on the crisp deterministic inputs and outputs requirement, which are the limited by an application in real world problems with both variation in term of randomness and vagueness of input and output data. Therefore the FSDEA-SC model is proposed to handle this problem and two steps of transforming the FSDEA-SC model into the CDDEA-SC model based on the concept of CC and the possibility approach are used in this paper.

The paper is organized as follows. The traditional DEA and the FSDEA-SC model are introduced in Second 2. The concept of CC and its deterministic equivalent are used to transform the FSDEA-SC into the fuzzy deterministic DEA model (FDDEA-SC) in Section 3. In Section 4, by using the concept of possibility approach, the FDDEA-SC model is transformed into the CDDEA-SC model. Finally, Section 5 concludes the paper.

2. Background

2.1. Data Envelopment Analysis (DEA) model

The first DEA model proposed by Charnes et al. (1978) is the CCR model which presented a constant return to scale (CRS) model for evaluating the performance of a set of comparable DMUs. In the usual setting, there are n evaluated DMUs, each of which consume the same type of m inputs and produce the same type of s outputs. If the input and output data for DMU_j for j = 1, …, n are $(x_{1j}, …, x_{mj})$ and $(y_{1j}, …, y_{sj})$, respectively, then the efficient of the evaluated target DMU (DMU_o) is measured by solving the dual problem of CCR model or envelopment model (DCCR) which is expressed with a real variable θ and λ_j for j = 1, …, n in (1)-(5).

(Phase I-DCCR) $\min \theta$ (1)

(Phase II-DCCR) $\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$ (2)

Subject to $\theta x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = 0$ (3)

$\sum_{j=1}^{n} \lambda_j y_{rj} - y_{ro} - s_r^+ = 0$ (4)

$\lambda_j \geq 0,\ s_i^- \geq 0,\ s_r^+ \geq 0$ (5)

Where (s_i^-, s_r^+) are slacks of input oriented envelopment model.

In the first of them, LP problem is solved to find optimal θ^* and find a solution that maximizes the sum of all slack with assurance that θ^* will not possible to improve any input or output with worsening some other input or output in second phase. The DMU_o is determined to be radial efficiency or CCR-inefficiency if and only if an optimal solution satisfies $\theta^* = 1$, and will be called CCR-efficiency if and only if an optimal solution satisfies $\theta^* = 1$ and all slacks are zero. An optimal solution $(\lambda_j^*, s_i^{-*}, s_r^{+*})$ from (1)-(5) is called the max-slack solution. If the max-slack solution satisfies both $s_i^{-*} = 0$ and $s_r^{+*} = 0$, then it is called zero slack. For an inefficient DMU_o, its efficiency of (x_{io}, y_{ro}) can be improved by projecting DMU_o into its reference set (E_o), which are defined by $E_o = \{j / \lambda_j^* > 0\}$ for $j \in \{1, 2, \ldots, n\}$. The projections of the input oriented DCCR model are given in the following formula.

$x_{io,improved} = \theta^* x_{io} - s_i^{-*}$ (6) $y_{ro,improved} = y_{ro} + s_r^{+*}$ (7)

Note that, there are three reasons for solving the envelopment model. First, the number of DMUs (n) is larger than the number of inputs and outputs (m + s) and hence it take to more time and larger memory to solve primal problem or multiplier model with n constraints than to solve the envelopment model with m + s constraints. Second, the inefficient DMU cannot be improved to improved activity because the reference set and max slack solution cannot be found in multiplier models. Finally, The interpretations of envelopment models are more straightforward than multiplier models. (Cooper *et al.* 2000)

2.2. FSDEA Model for Evaluating Supply Chain Performance

Suppose that there are n supply chains (DMUs) are evaluated in real world problems with both variations in term of randomness and vagueness of input and output data, each of which consume the same type of m fuzzy stochastic inputs and produce the same type of s fuzzy stochastic outputs. Let fuzzy stochastic direct input and output data of DMU_j are $(\tilde{\hat{x}}_{1j}^{Sup}, \ldots, \tilde{\hat{x}}_{mj}^{Sup}, \tilde{\hat{x}}_{1j}^{Ma}, \ldots, \tilde{\hat{x}}_{mj}^{Ma}, \tilde{\hat{x}}_{1j}^{Dis}, \ldots, \tilde{\hat{x}}_{mj}^{Dis}, \tilde{\hat{x}}_{1j}^{Re}, \ldots, \tilde{\hat{x}}_{mj}^{Re})$ and $(\tilde{\hat{y}}_{1j}^{Sup}, \ldots, \tilde{\hat{y}}_{mj}^{Sup}, \tilde{\hat{y}}_{1j}^{Ma}, \ldots, \tilde{\hat{y}}_{mj}^{Ma}, \tilde{\hat{y}}_{1j}^{Dis}, \ldots, \tilde{\hat{y}}_{mj}^{Dis}, \tilde{\hat{y}}_{1j}^{Re}, \ldots, \tilde{\hat{y}}_{mj}^{Re})$, respectively, $\tilde{\hat{z}}_{aj}^{SupMa}$ for a = 1, …, A; $\tilde{\hat{z}}_{bj}^{MaSup}$ for b = 1, …, B; $\tilde{\hat{z}}_{cj}^{MaDis}$ for c = 1, …, C; $\tilde{\hat{z}}_{dj}^{DisMa}$ for d = 1, …, D; $\tilde{\hat{z}}_{ej}^{MaRe}$ for e = 1, …, E; $\tilde{\hat{z}}_{fj}^{ReMa}$ for f = 1, …, F; $\tilde{\hat{z}}_{gj}^{DisRe}$ for g = 1, …, G; $\tilde{\hat{z}}_{hj}^{ReDis}$ for h = 1, …, H respectively represent fuzzy stochastic intermediate output data, therefore the model for evaluating supply chain performance is given in the following formula.

$$\text{(FSDEA-SC)}\ \min \Omega^{\Delta} = \frac{w_1\Omega^{Sup} + w_2\Omega^{Ma} + w_3\Omega^{Dis} + w_4\Omega^{Re}}{(w_1 + w_2 + w_3 + w_4)} \tag{8}$$

Subject to

$$\sum_{j=1}^{n}\lambda_j \tilde{\hat{x}}_{ij}^{Sup} - \Omega^{Sup}\tilde{\hat{x}}_{io}^{Sup} \le 0 \tag{9}$$

$$\tilde{\hat{y}}_{ro}^{Sup} - \sum_{j=1}^{n}\lambda_j \tilde{\hat{y}}_{rj}^{Sup} \le 0 \tag{10}$$

$$\breve{z}_{ao}^{SupMa} - \sum_{j=1}^{n}\lambda_j \tilde{\hat{z}}_{aj}^{SupMa} \le 0 \tag{11}$$

$$\sum_{j=1}^{n}\lambda_j \tilde{\hat{z}}_{bj}^{MaSup} - \breve{z}_{bo}^{MaSup} \le 0 \tag{12}$$

$$\sum_{j=1}^{n}\beta_j \tilde{\hat{x}}_{ij}^{Ma} - \Omega^{Ma}\tilde{\hat{x}}_{io}^{Ma} \le 0 \tag{13}$$

$$\tilde{\hat{y}}_{ro}^{Ma} - \sum_{j=1}^{n}\beta_j \tilde{\hat{y}}_{rj}^{Ma} \le 0 \tag{14}$$

$$\sum_{j=1}^{n}\beta_j \tilde{\hat{z}}_{aj}^{SupMa} - \breve{z}_{ao}^{SupMa} \le 0 \tag{15}$$

$$\breve{z}_{bo}^{MaSup} - \sum_{j=1}^{n}\beta_j \tilde{\hat{z}}_{bj}^{MaSup} \le 0 \tag{16}$$

$$\breve{z}_{co}^{MaDis} - \sum_{j=1}^{n}\beta_j \tilde{\hat{z}}_{cj}^{MaDis} \le 0 \tag{17}$$

$$\sum_{j=1}^{n}\beta_j \tilde{\hat{z}}_{dj}^{DisMa} - \breve{z}_{do}^{DisMa} \le 0 \tag{18}$$

$$\breve{z}_{eo}^{MaRe} - \sum_{j=1}^{n}\beta_j \tilde{\hat{z}}_{ej}^{MaRe} \le 0 \tag{19}$$

$$\sum_{j=1}^{n}\beta_j \tilde{\hat{z}}_{fj}^{ReMa} - \breve{z}_{fo}^{ReMa} \le 0 \tag{20}$$

$$\sum_{j=1}^{n}\chi_j \tilde{\hat{x}}_{ij}^{Dis} - \Omega^{Dis}\tilde{\hat{x}}_{io}^{Dis} \le 0 \tag{21}$$

$$\tilde{\hat{y}}_{ro}^{Dis} - \sum_{j=1}^{n}\chi_j \tilde{\hat{y}}_{rj}^{Dis} \le 0 \tag{22}$$

$$\sum_{j=1}^{n}\chi_j \tilde{\hat{z}}_{cj}^{MaDis} - \breve{z}_{co}^{MaDis} \le 0 \tag{23}$$

$$\breve{z}_{do}^{DisMa} - \sum_{j=1}^{n}\chi_j \tilde{\hat{z}}_{dj}^{DisMa} \le 0 \tag{24}$$

$$\breve{z}_{go}^{DisRe} - \sum_{j=1}^{n}\chi_j \tilde{\hat{z}}_{gj}^{DisRe} \le 0 \tag{25}$$

$$\sum_{j=1}^{n}\chi_j \tilde{\hat{z}}_{hj}^{ReDis} - \breve{z}_{ho}^{ReDis} \le 0 \tag{26}$$

$$\sum_{j=1}^{n}\delta_j \tilde{\hat{x}}_{ij}^{Re} - \Omega^{Re}\tilde{\hat{x}}_{io}^{Re} \le 0 \tag{27}$$

$$\tilde{\hat{y}}_{ro}^{Re} - \sum_{j=1}^{n}\delta_j \tilde{\hat{y}}_{rj}^{Re} \le 0 \tag{28}$$

$$\sum_{j=1}^{n}\delta_j \tilde{\hat{z}}_{ej}^{MaRe} - \breve{z}_{eo}^{MaRe} \le 0 \tag{29}$$

$$\breve{z}_{fo}^{ReMa} - \sum_{j=1}^{n}\delta_j \tilde{\hat{z}}_{fj}^{ReMa} \le 0 \tag{30}$$

$$\sum_{j=1}^{n}\delta_j \tilde{\hat{z}}_{gj}^{DisRe} - \breve{z}_{go}^{DisRe} \le 0 \tag{31}$$

$$\breve{z}_{ho}^{ReDis} - \sum_{j=1}^{n}\delta_j \tilde{\hat{z}}_{hj}^{ReDis} \le 0 \tag{32}$$

$$\Omega^{\Delta} \ge 0,\ \lambda_j \ge 0,\ \beta_j \ge 0,\ \chi_j \ge 0,\ \delta_j \ge 0 \tag{33}$$

Where Ω^{Δ} is supply chain efficiency, w are the user-specified weights reflecting the preference over performance of supply chain member, λ_j, β_j, χ_j and δ_j for j = 1, …, n are decision variables and "$\breve{\bullet}$" represent unknown decision variables.

3. The Equivalent Fuzzy Deterministic DEA for Supply Chain Evaluation

3.1. FSDEA Model for Evaluates Supply Chain Performance

In this section, the concept of CC which was introduced by Charnes and Cooper (1959) are used to transform model FSDDEA-SC to be FDDEA-SC. CC is a kind of stochastic optimization approaches. It is suitable for solving optimization problems with random variables. The constraints are guaranteed to be satisfied with a specified probability at the optimal solution found. Since constraints (8)-(33) are classified to be four types for each of supply chain's member constraint, i.e., direct input, direct output, intermediate input and intermediate output constraints. For example, there are four types for supplier's constraints in (9)-(12) and which are converted to be probability programming by CC.

Direct input; $\Pr\left\{\sum_{j=1}^{n}\lambda_j\tilde{\hat{x}}_{ij}^{Sup} - \Omega^{Sup}\tilde{\hat{x}}_{io}^{Sup} \le 0\right\} \ge 1 - \rho_i^{Sup}$ for $i \in DI^{Sup}$ (34)

Direct output; $\Pr\left\{\tilde{\hat{y}}_{ro}^{Sup} - \sum_{j=1}^{n}\lambda_j\tilde{\hat{y}}_{rj}^{Sup} \le 0\right\} \ge 1 - \tau_r^{Sup}$ for $r \in DR^{Sup}$ (35)

Intermediate output; $\Pr\left\{\breve{z}_{ao}^{SupMa} - \sum_{j=1}^{n}\lambda_j\tilde{\hat{z}}_{aj}^{SupMa} \le 0\right\} \ge 1 - \theta_a^{SupMa}$ (36)

Intermediate input; $\Pr\left\{\sum_{j=1}^{n}\lambda_j\tilde{\hat{z}}_{bj}^{MaSup} - \breve{z}_{bo}^{MaSup} \le 0\right\} \ge 1 - \theta_b^{MaSup}$ (37)

where "Pr" means probability, $1-\rho_i^{Sup}$, $1-\tau_r^{Sup}$, $1-\theta_a^{SupMa}$ and $1-\theta_b^{MaSup}$ are specified probability. "$\tilde{\hat{\bullet}}$" is a fuzzy random variable.

3.2. Deterministic Equivalent

Let ϖ_i, ω_r, κ_a^{SM}, κ_b^{MS} are external slack which can be inserted in the inequality outside braces to achieve equality, then supplier's probability equations are given in the following formula.

$$\Pr\left\{\sum_{j=1}^{n}\lambda_j\tilde{\hat{x}}_{ij}^{Sup} - \Omega^{Sup}\tilde{\hat{x}}_{io}^{Sup} \le 0\right\} = (1-\rho_i^{Sup}) + \varpi_i \quad (38)$$

$$\Pr\left\{\tilde{\hat{y}}_{ro}^{Sup} - \sum_{j=1}^{n}\lambda_j\tilde{\hat{y}}_{rj}^{Sup} \le 0\right\} = (1-\tau_r^{Sup}) + \omega_r \quad (39)$$

$$\Pr\left\{-\sum_{j=1}^{n}\lambda_j\tilde{\hat{z}}_{aj}^{SM} \le -\breve{z}_{ao}^{SM}\right\} = (1-\theta_a^{SM}) + \kappa_a^{SM} \quad (40)$$

$$\Pr\left\{\sum_{j=1}^{n}\lambda_j\tilde{\hat{z}}_{bj}^{MS} \le \breve{z}_{bo}^{MS}\right\} = (1-\theta_b^{MS}) + \kappa_b^{MS} \quad (41)$$

Since $\Pr\{\hat{\bullet} \le \Xi\} = (1-\vartheta) + \Gamma$ is equivalent with $\Pr\{\hat{\bullet} \le \Xi - S\} = 1-\vartheta$, where "$\hat{\bullet}$" is a random variable, Ξ is a constant, $1-\vartheta$ is a specified probability, Γ is an external slack and S is a positive number. If $\hat{\bullet}$ is assumed to be normal variable, then $\Pr\{\hat{\bullet} \le \Xi - S\} = 1-\vartheta$ is normalized by

$$\Pr\left\{\frac{\hat{\bullet} - E(\hat{\bullet})}{\sqrt{\sigma(\hat{\bullet})}} \le \frac{\Xi - S - E(\hat{\bullet})}{\sqrt{\sigma(\hat{\bullet})}}\right\} = 1-\vartheta \text{ or } -\Xi + S + E(\hat{\bullet}) = \Phi^{-1}(\vartheta)\sqrt{\sigma(\hat{\bullet})} \quad (42)$$

Where $E(\hat{\bullet})$ is expected value of random variable, $\sqrt{\sigma(\hat{\bullet})}$ is standard deviation of random variable and Φ represents the normal cumulative distribution function and Φ^{-1} is its inverse. In a manner similar with (42), deterministic equivalent of (38)-(41) are transformed to be

$$\sum_{j=1}^{n}\lambda_j E(\tilde{\hat{x}}_{ij}^{Sup}) - \Omega^{Sup}E(\tilde{\hat{x}}_{io}^{Sup}) + s_i^- = \Phi^{-1}(\rho_i^{Sup})\sqrt{[\Omega^{Sup}\lambda]^T \mathrm{Cov}_i[\Omega^{Sup}\lambda]} \quad (43)$$

$$E(\tilde{\hat{y}}_{ro}^{Sup}) - \sum_{j=1}^{n}\lambda_j E(\tilde{\hat{y}}_{rj}^{Sup}) + s_r^+ = \Phi^{-1}(\tau_r^{Sup})\sqrt{[1\lambda]^T \mathrm{Cov}_r[1\lambda]} \quad (44)$$

$$\breve{z}_{ao}^{SupMa} - \sum_{j=1}^{n}\lambda_j E(\tilde{\hat{z}}_{aj}^{SupMa}) + s_a^+ = \Phi^{-1}(\theta_a^{SupMa})\sqrt{[\lambda]^T \mathrm{Cov}_a[\lambda]} \quad (45)$$

$$\sum_{j=1}^{n}\lambda_j E(\tilde{\hat{z}}_{bj}^{MaSup}) - \breve{z}_{bo}^{MaSup} + s_b^- = \Phi^{-1}(\theta_b^{MaSup})\sqrt{[\lambda]^T \mathrm{Cov}_b[\lambda]} \quad (46)$$

Where s_i^-, s_r^+, s_a^+ and s_b^- are positive number, $[\theta\lambda] = (\theta, \lambda_1, \lambda_2, \ldots, \lambda_n)^T$, $[1\lambda] = (1, \lambda_1, \lambda_2, \ldots, \lambda_n)^T$, $[\lambda] = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$, Cov_i, Cov_r, Cov_a and Cov_b are in form

$$Cov_i = \begin{pmatrix} V(\tilde{\hat{x}}_{io}) & \cdots & Cov(\tilde{\hat{x}}_{io}, \tilde{\hat{x}}_{in}) \\ \vdots & \ddots & \vdots \\ Cov(\tilde{\hat{x}}_{io}, \tilde{\hat{x}}_{in}) & \cdots & V(\tilde{\hat{x}}_{in}) \end{pmatrix} \quad (47) \qquad Cov_r = \begin{pmatrix} V(\tilde{\hat{y}}_{ro}) & \cdots & Cov(\tilde{\hat{y}}_{ro}, \tilde{\hat{y}}_{rn}) \\ \vdots & \ddots & \vdots \\ Cov(\tilde{\hat{y}}_{ro}, \tilde{\hat{y}}_{rn}) & \cdots & V(\tilde{\hat{y}}_{rn}) \end{pmatrix} \quad (48)$$

$$Cov_a = \begin{pmatrix} V(\tilde{\hat{z}}_{a1}) & \cdots & Cov(\tilde{\hat{z}}_{a1}, \tilde{\hat{z}}_{an}) \\ \vdots & \ddots & \vdots \\ Cov(\tilde{\hat{z}}_{a1}, \tilde{\hat{z}}_{an}) & \cdots & V(\tilde{\hat{z}}_{an}) \end{pmatrix} \quad (49) \qquad Cov_b = \begin{pmatrix} V(\tilde{\hat{z}}_{b1}) & \cdots & Cov(\tilde{\hat{z}}_{b1}, \tilde{\hat{z}}_{bn}) \\ \vdots & \ddots & \vdots \\ Cov(\tilde{\hat{z}}_{b1}, \tilde{\hat{z}}_{bn}) & \cdots & V(\tilde{\hat{z}}_{bn}) \end{pmatrix} \quad (50)$$

Where symbol "V" and "Cov" refers to a variance and covariance operator, respectively. To obtain the deterministic equivalent to (43)-(46), the linearization approach to obtain a linear deterministic equivalent, which was introduced by Cooper *et al.* (1998) is used. In this approach, stochastic variables ($\tilde{\hat{x}}_{io}^{Sup}$, $\tilde{\hat{x}}_{ij}^{Sup}$, $\tilde{\hat{y}}_{ro}^{Sup}$, $\tilde{\hat{y}}_{rj}^{Sup}$, $\tilde{\hat{z}}_{aj}^{SupMa}$ and $\tilde{\hat{z}}_{bj}^{MaSup}$) are assumed to be

$$\tilde{\hat{x}}_{io}^{Sup} = \tilde{\bar{x}}_{io}^{Sup} + \tilde{p}_{io}^{Sup}\zeta_{io}\,;\; \tilde{\hat{y}}_{ro}^{Sup} = \tilde{\bar{y}}_{ro}^{Sup} + \tilde{q}_{ro}^{Sup}\xi_{ro}\,; \quad (51)$$

$$\tilde{\hat{x}}_{ij}^{Sup} = \tilde{\bar{x}}_{ij}^{Sup} + \tilde{p}_{ij}^{Sup}\zeta_{ij}\,;\; \tilde{\hat{y}}_{rj}^{Sup} = \tilde{\bar{y}}_{rj}^{Sup} + \tilde{q}_{rj}^{Sup}\xi_{rj}\,; \quad (52)$$

$$\tilde{\hat{z}}_{aj}^{SupMa} = \tilde{\bar{z}}_{aj}^{SupMa} + \tilde{k}_{aj}^{SupMa}\varsigma_{aj}\,;\; \tilde{\hat{z}}_{bj}^{MaSup} = \tilde{\bar{z}}_{bj}^{MaSup} + \tilde{k}_{bj}^{SupMa}\varsigma_{bj} \quad (53)$$

Where $\tilde{\bar{x}}_{io}^{Sup}$, $\tilde{\bar{y}}_{ro}^{Sup}$, $\tilde{\bar{x}}_{ij}^{Sup}$, $\tilde{\bar{y}}_{rj}^{Sup}$, $\tilde{\bar{z}}_{aj}^{SupMa}$ and $\tilde{\bar{z}}_{bj}^{MaSup}$ are the fuzzy mean. $\tilde{p}_{io}^{Sup}\zeta_{io}$, $\tilde{q}_{ro}^{Sup}\xi_{ro}$, $\tilde{p}_{ij}^{Sup}\zeta_{ij}$, $\tilde{q}_{rj}^{Sup}\xi_{rj}$, $\tilde{k}_{aj}^{SupMa}\varsigma_{aj}$ and $\tilde{k}_{bj}^{SupMa}\varsigma_{bj}$ are error terms. Since $\tilde{p}_{io}^{Sup}$, $\tilde{q}_{ro}^{Sup}$, $\tilde{p}_{ij}^{Sup}$, $\tilde{q}_{rj}^{Sup}$, $\tilde{k}_{aj}^{SupMa}$ and $\tilde{k}_{bj}^{SupMa}$ are the fuzzy standard deviation and ζ_{io}, ξ_{ro}, ζ_{ij}, ξ_{rj}, ς_{aj} and ς_{bj} are error structure which are assumed to be normal distribution $N(0, \sigma^2)$, therefore the fuzzy expected value and variance of variables are given in the following formula.

$$E(\tilde{\hat{x}}_{io}^{Sup}) = \tilde{\bar{x}}_{io}^{Sup},\; E(\tilde{\hat{y}}_{ro}^{Sup}) = \tilde{\bar{y}}_{ro}^{Sup},\; E(\tilde{\hat{x}}_{ij}^{Sup}) = \tilde{\bar{x}}_{ij}^{Sup},$$

$$E(\tilde{\hat{y}}_{rj}^{Sup}) = \tilde{\bar{y}}_{rj}^{Sup},\; E(\tilde{\hat{z}}_{aj}^{SupMa}) = \tilde{\bar{z}}_{aj}^{SupMa},\; E(\tilde{\hat{z}}_{bj}^{MaSup}) = \tilde{\bar{z}}_{bj}^{MaSup} \quad (54)$$

$$Var(\tilde{\hat{x}}_{io}^{Sup}) = (\tilde{p}_{io}^{Sup}\sigma)^2,\; Var(\tilde{\hat{y}}_{ro}^{Sup}) = (\tilde{q}_{ro}^{Sup}\sigma)^2,\; Var(\tilde{\hat{x}}_{ij}^{Sup}) = (\tilde{p}_{ij}^{Sup}\sigma)^2,$$

$$Var(\tilde{\hat{y}}_{rj}^{Sup}) = (\tilde{q}_{rj}^{Sup}\sigma)^2,\; Var(\tilde{\hat{z}}_{aj}^{SupMa}) = (\tilde{k}_{aj}^{SupMa}\sigma)^2,\; Var(\tilde{\hat{z}}_{bj}^{MaSup}) = (\tilde{k}_{bj}^{SupMa}\sigma)^2 \quad (55)$$

Since Cov_i, Cov_r, Cov_a and Cov_b are diagonal matrix, then

$$\sqrt{[\Omega^{Sup}\lambda]^T Cov_i [\Omega^{Sup}\lambda]} = \Omega^{Sup}\tilde{p}_{io}^{Sup}\sigma + \sum_{j=1}^{n}\lambda_j\tilde{p}_{ij}^{Sup}\sigma \quad (56)$$

$$\sqrt{[1\lambda]^T Cov_r [1\lambda]} = \tilde{q}_{ro}^{Sup}\sigma + \sum_{j=1}^{n}\lambda_j\tilde{q}_{rj}^{Sup}\sigma \quad (57)$$

$$\sqrt{[\lambda]^T Cov_a [\lambda]} = \sum_{j=1}^{n}\lambda_j\tilde{k}_{aj}^{SupMa}\sigma \quad (58) \qquad \sqrt{[\lambda]^T Cov_b [\lambda]} = \sum_{j=1}^{n}\lambda_j\tilde{k}_{bj}^{MaSup}\sigma \quad (59)$$

Therefore, (43)-(46) are reformulated to be the following formula.

$$\sum_{j=1}^{n}\lambda_j\tilde{\bar{x}}_{ij}^{Sup} - \Omega^{Sup}\tilde{\bar{x}}_{io}^{Sup} + s_i^- = \Phi^{-1}(\rho_i^{Sup})\left(\Omega^{Sup}\tilde{p}_{io}^{Sup}\sigma + \sum_{j=1}^{n}\lambda_j\tilde{p}_{ij}^{Sup}\sigma\right) \quad (60)$$

$$\tilde{\tilde{y}}_{ro}^{Sup} - \sum_{j=1}^{n}\lambda_j \tilde{\tilde{y}}_{rj}^{Sup} + s_r^{+} = \Phi^{-1}(\tau_r^{Sup})\left(\tilde{q}_{ro}^{Sup}\sigma + \sum_{j=1}^{n}\lambda_j \tilde{q}_{rj}^{Sup}\sigma\right) \quad (61)$$

$$\breve{z}_{ao}^{SupMa} - \sum_{j=1}^{n}\lambda_j \tilde{\breve{z}}_{aj}^{SupMa} + s_a^{+} = \Phi^{-1}(\theta_a^{SupMa})\left(\sum_{j=1}^{n}\lambda_j \tilde{k}_{aj}^{SupMa}\sigma\right) \quad (62)$$

$$\sum_{j=1}^{n}\lambda_j \tilde{\breve{z}}_{bj}^{MaSup} - \breve{z}_{bo}^{MaSup} + s_b^{-} = \Phi^{-1}(\theta_b^{MaSup})\left(\sum_{j=1}^{n}\lambda_j \tilde{k}_{bj}^{MaSup}\sigma\right) \quad (63)$$

4. The Equivalent Crisp Deterministic DEA for Supply Chain Evaluation

4.1. Possibility Approach

Possibility theory in the context of the fuzzy set theory was introduced by Zadeh (1978) which is dealing with non-stochastic imprecision and vagueness. A good reference on possibility theory is Dubois and Prade (1980) and Zimmermann (1996). Suppose that $(\Theta_i, P(\Theta_i), \pi_i)$ for i = 1, …, n are a possibility space with Θ_i being the nonempty set of interest, $P(\Theta_i)$ is the collection of all subset of Θ_i, and π_i is the possibility measure from $P(\Theta_i)$ to [0, 1], then $\pi(\phi) = 0$ and $\pi(\Theta_i) = 1$, $\pi(\cup_i A_i) = \sup_i\{\pi(A_i)\}$ with each $A_i \in P(\Theta_i)$. Let $\tilde{\psi}$ be fuzzy variable as a real-valued function defined over Θ_i with the membership function $\mu_{\tilde{\psi}}(s)$ as

$$\mu_{\tilde{\psi}}(s) = \sup_{\theta_i \in \Theta_i} \{\pi\{(\theta_i)\} / \tilde{\psi}(\theta_i) = s\} \text{ for } s \in R \quad (64)$$

From possibility theory (Zadeh, 1978), if (Θ, P(Θ), π) be a product possibility space such that $\Theta = \Theta_{1,\ldots,}\Theta_n$ then

$$\pi(A) = \min\{\pi_i(A_i) / A = A_1 \times \ldots \times A_n, A_i \in P(\Theta_i))\quad (65)$$

To compare two fuzzy variables (Dubois and Prade, 1980), let $\tilde{a}$ and $\tilde{b}$ are two fuzzy variables on the possibility spaces $(\Theta_1, P(\Theta_1), \pi_1)$ and $(\Theta_2, P(\Theta_2), \pi_2)$, respectively, then the possibility measure of fuzzy event $\tilde{a} \le \tilde{b}$ is defined on the product possibility space ($\Theta = \Theta_1 \times \Theta_2$, P(Θ), π) by

$$\pi(\tilde{a} \le \tilde{b}) = = \sup_{s,t \in R}\{\min\{\mu_{\tilde{a}}(s) \le \mu_{\tilde{b}}(t)\} / s \le t\} \quad (66)$$

Let $\tilde{a}_1, \ldots, \tilde{a}_n$ be fuzzy variables and $f_j : R^n \to R$ be a real-valued function for j = 1, …, m. The possibility measure of fuzzy event is given by

$$\pi(f_j(\tilde{a}_1, \ldots, \tilde{a}_n) \le 0) = \sup_{s_1, \ldots, s_n \in R}\{\min\{\mu_{\tilde{a}_i}(s_i)\} / f_j(s_1, \ldots, s_n) \le 0\} \text{ for } j = 1, \ldots, m \quad (67)$$

Lertworasirikul *et al.* (2003) was proposed and proofed Lemma 1 and its can be adopted to transform the fuzzy DEA to be crisp DEA model and which is used to transform the fuzzy stochastic CCR in multiplier form to be crisp deterministic CCR by Punyangarm *et al.* (2006)

Lemma 1. Let $\tilde{a}_1, \ldots, \tilde{a}_n$ be fuzzy variables with normal and convex membership functions. Let $(\bullet)_{\alpha_i}^{L}$ and $(\bullet)_{\alpha_i}^{U}$ denote the lower and upper bounds of the α-level set of $\tilde{a}_i$; i = 1, …, n for possibility levels α_1, α_2 and α_3 with $0 \le \alpha_1, \alpha_2, \alpha_3 \le 1$, then

(i) $\pi(\tilde{a}_1 + \cdots + \tilde{a}_n \le b) \ge \alpha_1$ iff $(\tilde{a}_1)_{\alpha_1}^{L} + \cdots + (\tilde{a}_n)_{\alpha_1}^{L} \le b$,

(ii) $\pi(\tilde{a}_1 + \cdots + \tilde{a}_n \ge b) \ge \alpha_2$ iff $(\tilde{a}_1)_{\alpha_2}^{U} + \cdots + (\tilde{a}_n)_{\alpha_2}^{U} \ge b$,

(iii) $\pi(\tilde{a}_1 + \cdots + \tilde{a}_n = b) \ge \alpha_3$ iff $(\tilde{a}_1)_{\alpha_3}^{L} + \cdots + (\tilde{a}_n)_{\alpha_3}^{L} \le b$ and $(\tilde{a}_1)_{\alpha_3}^{U} + \cdots + (\tilde{a}_n)_{\alpha_3}^{U} \ge b$

4.2. Crisp Deterministic Equivalence

From lemma 1, let α_i for i = 1, …, m; α_r for r = 1, …, s; α_a for a = 1, …, A and α_b for b = 1, …, B respectively represent possibility levels of direct input, output and intermediate input (output) data. By the concept FDDEA model becomes the following supplier's constraints of possibility deterministic DEA.

$$\mathrm{Pos}\left\{\Phi^{-1}(\rho_i^{Sup})\left(\Omega^{Sup}\tilde{p}_{io}^{Sup}\sigma+\sum_{j=1}^{n}\lambda_j\tilde{p}_{ij}^{Sup}\sigma\right)-\sum_{j=1}^{n}\lambda_j\tilde{\tilde{x}}_{ij}^{Sup}+\Omega^{Sup}\tilde{\tilde{x}}_{io}^{Sup}=s_i^-\right\}\geq\alpha_i \quad (68)$$

$$\mathrm{Pos}\left\{\Phi^{-1}(\tau_r^{Sup})\left(\tilde{q}_{ro}^{Sup}\sigma+\sum_{j=1}^{n}\lambda_j\tilde{q}_{rj}^{Sup}\sigma\right)-\tilde{\tilde{y}}_{ro}^{Sup}+\sum_{j=1}^{n}\lambda_j\tilde{\tilde{y}}_{rj}^{Sup}=s_r^+\right\}\geq\alpha_r \quad (69)$$

$$\mathrm{Pos}\left\{\Phi^{-1}(\theta_a^{SM})\left(\sum_{j=1}^{n}\lambda_j\tilde{k}_{aj}^{SM}\sigma\right)+\sum_{j=1}^{n}\lambda_j\tilde{\tilde{z}}_{aj}^{SM}=\tilde{z}_{ao}^{SM}+s_a^+\right\}\geq\alpha_a \quad (70)$$

$$\mathrm{Pos}\left\{\Phi^{-1}(\theta_b^{MS})\left(\sum_{j=1}^{n}\lambda_j\tilde{k}_{bj}^{MS}\sigma\right)-\sum_{j=1}^{n}\lambda_j\tilde{\tilde{z}}_{bj}^{MS}=s_b^- -\tilde{z}_{bo}^{MS}\right\}\geq\alpha_b \quad (71)$$

where "Pos" means possibility. The equivalent fuzzy deterministic DEA model of supplier's constraints in (68)-(71) are transformed to be crisp deterministic DEA (CDDEA) model as

$$\Phi^{-1}(\rho_i^{Sup})\left(\Omega^{Sup}(\tilde{p}_{io}^{Sup})_{\alpha_i}^{L}\sigma+\sum_{j=1}^{n}\lambda_j(\tilde{p}_{ij}^{Sup})_{\alpha_i}^{L}\sigma\right)-\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{x}}_{ij}^{Sup})_{\alpha_i}^{L}+\Omega^{Sup}(\tilde{\tilde{x}}_{io}^{Sup})_{\alpha_i}^{L}\leq s_i^- \quad (72)$$

$$\Phi^{-1}(\rho_i^{Sup})\left(\Omega^{Sup}(\tilde{p}_{io}^{Sup})_{\alpha_i}^{U}\sigma+\sum_{j=1}^{n}\lambda_j(\tilde{p}_{ij}^{Sup})_{\alpha_i}^{U}\sigma\right)-\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{x}}_{ij}^{Sup})_{\alpha_i}^{U}+\Omega^{Sup}(\tilde{\tilde{x}}_{io}^{Sup})_{\alpha_i}^{U}\geq s_i^- \quad (73)$$

$$\Phi^{-1}(\tau_r^{Sup})\left((\tilde{q}_{ro}^{Sup})_{\alpha_r}^{L}\sigma+\sum_{j=1}^{n}\lambda_j(\tilde{q}_{rj}^{Sup})_{\alpha_r}^{L}\sigma\right)-(\tilde{\tilde{y}}_{ro}^{Sup})_{\alpha_r}^{L}+\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{y}}_{rj}^{Sup})_{\alpha_r}^{L}\leq s_r^+ \quad (74)$$

$$\Phi^{-1}(\tau_r^{Sup})\left((\tilde{q}_{ro}^{Sup})_{\alpha_r}^{U}\sigma+\sum_{j=1}^{n}\lambda_j(\tilde{q}_{rj}^{Sup})_{\alpha_r}^{U}\sigma\right)-(\tilde{\tilde{y}}_{ro}^{Sup})_{\alpha_r}^{U}+\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{y}}_{rj}^{Sup})_{\alpha_r}^{U}\geq s_r^+ \quad (75)$$

$$\Phi^{-1}(\theta_a^{SM})\left(\sum_{j=1}^{n}\lambda_j(\tilde{k}_{aj}^{SM})_{\alpha_a}^{L}\sigma\right)+\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{z}}_{aj}^{SM})_{\alpha_a}^{L}\leq\tilde{z}_{ao}^{SM}+s_a^+ \quad (76)$$

$$\Phi^{-1}(\theta_a^{SM})\left(\sum_{j=1}^{n}\lambda_j(\tilde{k}_{aj}^{SM})_{\alpha_a}^{U}\sigma\right)+\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{z}}_{aj}^{SM})_{\alpha_a}^{U}\geq\tilde{z}_{ao}^{SM}+s_a^+ \quad (77)$$

$$\Phi^{-1}(\theta_b^{MS})\left(\sum_{j=1}^{n}\lambda_j(\tilde{k}_{bj}^{MS})_{\alpha_b}^{L}\sigma\right)-\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{z}}_{bj}^{MS})_{\alpha_b}^{L}\leq s_b^- -\tilde{z}_{bo}^{MS} \quad (78)$$

$$\Phi^{-1}(\theta_b^{MS})\left(\sum_{j=1}^{n}\lambda_j(\tilde{k}_{bj}^{MS})_{\alpha_b}^{U}\sigma\right)-\sum_{j=1}^{n}\lambda_j(\tilde{\tilde{z}}_{bj}^{MS})_{\alpha_b}^{U}\geq s_b^- -\tilde{z}_{bo}^{MS} \quad (79)$$

Similar to the transforming crisp deterministic equivalent procedures of supplier's constraints in (34)-(37) to (72)-(79), the other supply chain's members are transformed to be crisp deterministic equivalent by the same procedures.

5. Conclusion

Differentiate with the performance evaluation by traditional DEA approach. The DEA model for supply chain performance evaluation is involved both of direct and intermediate input-output system. Since the effect of randomness and vagueness variation of input and output data, then the two steps of transform procedure which are chance constraints and possibility approach is used to convert the FSDEA to be linear programming which is referred as CDDEA model. This procedure helps us to delimiting the problem with fuzzy and stochastic inputs and outputs. However, the effect of this procedure threats the number of new model's constraints to be increased from (m + s + A + … + H) constraints to 2(m + s + A + … + H) constraints.

6. Reference

A. Charnes, W.W. Cooper and E. Rhodes (1978) Measuring the efficiency of decision making units, *European Journal of Operational Research*, 2, 429-444.

A. Charnes and W.W. Cooper (1959) Chance constrained programming, *Management Science*, 6(1), 73-79.

W.W. Cooper, Z.M. Huang, V. Lelas, S.X Li and O.B. Olesen (1998) Chance constrained programming formulations for stochastic characterizations of efficiency and dominance in DEA, *Journal of Productivity Analysis*, 9, 53-79.

W.W. Cooper, Z.M. Huang and S.X. Li (1996) Satisficing DEA models under chance constraints, *Annals of Operations Research*, 66, 279-295.

W.W. Cooper, L.M. Seiford and K. Tone (2000) *Data envelopment analysis: A comprehensive text with models, applications, reference and DEA-solver software*, Kluwer Academic Publishers, London.

D. Dubois, H. Prade (1980) *Fuzzy sets and systems: Theory and applications*, Academic Press, New York.

S. Lertworasirikul, S.C. Fang, J. A. Joines and H.L.W. Nuttle (2003) Fuzzy data envelopment analysis (DEA): A possibility approach, *Fuzzy Sets and Systems*, 139, 379-394.

V. Punyangarm, P. Charnsethikul, S. Lertworasirikul and P. Yanpirat (2006) Fuzzy Stochastic Data Envelopment Analysis (FSDEA) Model: A Possibility Approach. *Proceedings of the Statistic and Applied Statistic Conference, Chonburi, Thailand,* C88-C93.

L.A. Zadeh (1978) Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems*, 1, 3-28.

J. Zhu (2003) *Quantitative models for performance evaluation and benchmarking: data envelopment analysis with spreadsheets and DEA excel solver*, Kluwer Academic Publishers, Boston

H.J. Zimmermann (1996) *Fuzzy Set Theory and Its Application*, Kluwer Academic Publishers, London.